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Numerical Solution of Nonlinear Problems with Multiple Roots Using Derivative-Free Algorithms

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Abstract: In the study of systems' dynamics the presence of symmetry dramatically reduces the complexity, while in chemistry, symmetry plays a central role in the analysis of the structure, bonding, and spectroscopy of molecules. In a more general context, the principle of equivalence, a principle of local symmetry, dictated the dynamics of gravity, of space-time itself. In certain instances, especially in the presence of symmetry, we end up having to deal with an equation with multiple roots. A variety of optimal methods have been proposed in the literature for multiple roots with known multiplicity, all of which need derivative evaluations in the formulations. However, in the literature, optimal methods without derivatives are few. Motivated by this feature, here we present a novel optimal family of fourth-order methods for multiple roots with known multiplicity, which do not use any derivative. The scheme of the new iterative family consists of two steps, namely Traub-Steffensen and Traub-Steffensen-like iterations with weight factor. According to the Kung-Traub hypothesis, the new algorithms satisfy the optimality criterion. Taylor's series expansion is used to examine order of convergence. We also demonstrate the application of new algorithms to real-life problems, i.e., Van der Waals problem, Manning problem, Planck law radiation problem, and Kepler's problem. Furthermore, the performance comparisons have shown that the given derivative-free algorithms are competitive with existing optimal fourth-order algorithms that require derivative information.

Keywords: multiple roots; convergence; nonlinear equations; derivative-free method

MSC: 49M15; 65H05; 41A25

Citation: Kumar, S.; Sharma, J.R.; Bhagwan, J.; Jäntschi, L. Numerical Solution of Nonlinear Problems with Multiple Roots Using Derivative-Free Algorithms. *Symmetry* **2023**, *15*, 1249. https://doi.org/10.3390/sym15061249

Academic Editor: Theodore E. Simos

Received: 5 May 2023
Revised: 5 June 2023
Accepted: 9 June 2023
Published: 12 June 2023

1. Introduction

Simple systems often embed a good amount of symmetry. Take for instance the characteristic polynomial (ChP) of hydrocarbons [1]. Considering 3 cases here, propane (ChP is $x^3 - 2x$), normal butane (ChP is $x^4 - 3x^2 + x$) and isobutane (ChP is $x^4 - 3x^2$), one should easily notice that the highest symmetry is in isobutane. At the same time, isobutane is the one having multiple roots in the characteristic polynomial. Same symmetry is responsible for the presence of the multiple roots in the ChP of 2,2,4,4-Tetramethylpentane (ChP is $x^9 - 8x^7 + 15x^5$, see a_25 in [2]). One should notice that, in the selected cases, the multiple root is banal ($x = 0$); however, in general, in more complex cases, the multiple root is not any more banal.

Much research has been conducted on the solution of nonlinear equations and systems of nonlinear equations. There are numerous publications on the topic, including those given in reference [3–10], and Traub's book [11] has a whole chapter devoted to

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The Numerical Solution of Nonlinear Problems Christopher T. H. Baker,Chris Phillips,1981 **Numerical solution of nonlinear problems** James M. Ortega,1970 *Numerical Solution of Nonlinear Equations* E.L. Allgöwer,K. Glashoff,H.-O. Peitgen,2006-11-14 *Numerical Solutions of Nonlinear Problems* James M. Ortega,Werner C. Rheinboldt,1970 **Newton Methods for Nonlinear Problems** Peter Deuflhard,2011-09-18 This book deals with the efficient numerical solution of challenging nonlinear problems in science and engineering both in finite dimension algebraic systems and in infinite dimension ordinary and partial differential equations Its focus is on local and global Newton methods for direct problems or Gauss Newton methods for inverse problems The term affine invariance means that the presented algorithms and their convergence analysis are invariant under one out of four subclasses of affine transformations of the problem to be solved Compared to traditional textbooks the distinguishing affine invariance approach leads to shorter theorems and proofs and permits the construction of fully adaptive algorithms Lots of numerical illustrations comparison tables and exercises make the text useful in computational mathematics classes At the same time the book opens many directions for possible future research Studies in Numerical Analysis 2 James McDonough Ortega,Werner Carl Rheinboldt,1970 Numerical Methods for Nonlinear Partial Differential Equations Sören Bartels,2015-01-19 The description of many interesting phenomena in science and engineering leads to infinite dimensional minimization or evolution problems that define nonlinear partial differential equations While the development and analysis of numerical methods for linear partial differential equations is nearly complete only few results are available in the case of nonlinear equations This monograph devises numerical methods for nonlinear model problems arising in the mathematical description of phase transitions large bending problems image processing and inelastic material behavior For each of these problems the underlying mathematical model is discussed the essential analytical properties are explained and the proposed numerical method is rigorously analyzed The practicality of the algorithms is illustrated by means of short implementations *Numerical Solutions of Nonlinear Problems* James M. Ortega,Werner C. Rheinboldt,1970 *Topics in Numerical Analysis* G. Alefeld,Xiaojun Chen,2001-09-11 This collection of papers on numerical analysis with special emphasis on nonlinear problems covers a broad spectrum of fields Several papers are involved in applying numerical methods for proving the existence of solutions of nonlinear problems e g of boundary problems or of obstacle problems Naturally the solution of linear and nonlinear problems by iterative methods is the subject of a couple of papers Here topics like the fast verification of solutions of monotone matrix equations the convergence of linear asynchronous iteration with spectral radius of modulus one or aggregation and disaggregation methods for p cyclic Markov chains are treated On the other hand papers involved in optimization problems can be found Nearly all fields of modern numerical analysis are touched by at least one paper **Numerical Solutions of Nonlinear Problems** James M. Ortega,Werner C. Rheinboldt,1970 **Numerical Methods for Unconstrained Optimization and**

Nonlinear Equations J. E. Dennis, Jr.,Robert B. Schnabel,1996-12-01 A complete state of the art description of the methods for unconstrained optimization and systems of nonlinear equations **Numerical Solution of Nonlinear Problems** James M. Ortega,1970 **Numerical Solution of Systems of Nonlinear Algebraic Equations** George D. Byrne,Charles A. Hall,2014-05-10 Numerical Solution of Systems of Nonlinear Algebraic Equations contains invited lectures of the NSF CBMS Regional Conference on the Numerical Solution of Nonlinear Algebraic Systems with Applications to Problems in Physics Engineering and Economics held on July 10 14 1972 This book is composed of 10 chapters and begins with the concepts of nonlinear algebraic equations in continuum mechanics The succeeding chapters deal with the numerical solution of quasilinear elliptic equations the nonlinear systems in semi infinite programming and the solution of large systems of linear algebraic equations These topics are followed by a survey of some computational techniques for the nonlinear least squares problem The remaining chapters explore the problem of nonlinear functional minimization the modification methods and the computer oriented algorithms for solving system These chapters also examine the principles of contractor theory of solving equations This book will prove useful to undergraduate and graduate students Studies in Numerical Analysis 2 Numerical Solutions of Nonlinear Problems a Collection of Papers Presented at Symposia in Numeial United States,1970 *Studies in Numerical Analysis 2* James McDonough Ortega,Werner Carl Rheinboldt,1970 *Computational Solution of Nonlinear Systems of Equations* Eugene L. Allgower,Kurt Georg,1990-04-03 Nonlinear equations arise in essentially every branch of modern science engineering and mathematics However in only a very few special cases is it possible to obtain useful solutions to nonlinear equations via analytical calculations As a result many scientists resort to computational methods This book contains the proceedings of the Joint AMS SIAM Summer Seminar Computational Solution of Nonlinear Systems of Equations held in July 1988 at Colorado State University The aim of the book is to give a wide ranging survey of essentially all of the methods which comprise currently active areas of research in the computational solution of systems of nonlinear equations A number of entry level survey papers were solicited and a series of test problems has been collected in an appendix Most of the articles are accessible to students who have had a course in numerical analysis *Numerical Methods for Nonlinear Engineering Models* John R. Hauser,2009-03-24 There are many books on the use of numerical methods for solving engineering problems and for modeling of engineering artifacts In addition there are many styles of such presentations ranging from books with a major emphasis on theory to books with an emphasis on applications The purpose of this book is hopefully to present a somewhat different approach to the use of numerical methods for gineering applications Engineering models are in general nonlinear models where the response of some appropriate engineering variable depends in a nonlinear manner on the plication of some independent parameter It is certainly true that for many types of engineering models it is sufficient to approximate the real physical world by some linear model However when engineering environments are pushed to treme conditions nonlinear effects are always encountered It is also such treme conditions that are of major

importance in determining the reliability or failure limits of engineering systems Hence it is essential than engineers have a toolbox of modeling techniques that can be used to model nonlinear engineering systems Such a set of basic numerical methods is the topic of this book For each subject area treated nonlinear models are incorporated into the discussion from the very beginning and linear models are simply treated as special cases of more general nonlinear models This is a basic and fundamental difference in this book from most books on numerical methods *Studies in Numerical Analysis* Society for Industrial and Applied Mathematics,1970 *Numerical Methods for Nonlinear Variational Problems* Roland Glowinski,2013-06-29 Many mechanics and physics problems have variational formulations making them appropriate for numerical treatment by finite element techniques and efficient iterative methods This book describes the mathematical background and reviews the techniques for solving problems including those that require large computations such as transonic flows for compressible fluids and the Navier Stokes equations for incompressible viscous fluids Finite element approximations and non linear relaxation augmented Lagrangians and nonlinear least square methods are all covered in detail as are many applications Numerical Methods for Nonlinear Variational Problems originally published in the Springer Series in Computational Physics is a classic in applied mathematics and computational physics and engineering This long awaited softcover re edition is still a valuable resource for practitioners in industry and physics and for advanced students *Numerical Solution of Nonlinear Boundary Value Problems with Applications* Milan Kubicek,Vladimir Hlavacek,2008-01-01 A survey of the development analysis and application of numerical techniques in solving nonlinear boundary value problems this text presents numerical analysis as a working tool for physicists and engineers Starting with a survey of accomplishments in the field it explores initial and boundary value problems for ordinary differential equations linear boundary value problems and the numerical realization of parametric studies in nonlinear boundary value problems The authors Milan Kubicek Professor at the Prague Institute of Chemical Technology and Vladimir Hlavacek Professor at the University of Buffalo emphasize the description and straightforward application of numerical techniques rather than underlying theory This approach reflects their extensive experience with the application of diverse numerical algorithms

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